4ª Lista de Exercícios de E.D.O

1-) Determine as transformadas de Laplace das funções definidas pelas seguintes expressões analíticas:

a-) $f(t) = e^{3t}\cos(2t)$

b-) $f(t) = \cos^2(at)$

c-) $f(t) = t^2 \operatorname{sen} t$

d-) $f(t) = t^3 e^{-t}$

e-) $f(t) = t e^{at}\cos(bt)$, $a, b \in \mathbb{R}$

Respostas:

a-) $F(s) = \dfrac{s-3}{(s-3)^2+4}$, b-) $F(s) = \dfrac{1}{2s} + \dfrac{s}{2(s^2+4a^2)}$

c-) $F(s) = \dfrac{6s^2-2}{(s^2+1)^3}$, d-) $F(s) = \dfrac{6}{(s+1)^4}$, e-) $F(s) = \dfrac{(s-a)^2-b^2}{[(s-a)^2+b^2]^2}$

2-) Determine a transformada de Laplace das seguintes funções:

a-) $f(t) = \dfrac{\operatorname{sen} t}{t}$, Resp: $F(s) = \dfrac{\pi}{2} - \operatorname{arc\,tg} s$, $s > 0$

b-) $g(t) = \dfrac{\cos(at) - 1}{t}$, Resp: $G(s) = -\ln \dfrac{\sqrt{s^2+a^2}}{s}$, $s > 0$

c-) $h(t) = \dfrac{e^{at} - e^{-at}}{t}$, Resp: $H(s) = \ln\left|\dfrac{s+a}{s-a}\right|$, $s > |a|$

3-) Calcule:

Respostas

a.) $\mathcal{L}^{-1}\left\{ (s-2)^{-2} \right\}$

a.) $t\,e^{2t}$

b.) $\mathcal{L}^{-1}\left\{ \dfrac{7}{(s-1)^3} + \dfrac{1}{(s+1)^2-4} \right\}$

b.) $\frac{7}{2} t^2 e^{t} + \frac{1}{2} e^{-t} \operatorname{senh}(2t)$

c.) $\mathcal{L}^{-1}\left\{ \dfrac{s}{(s+1)^2(s^2+1)} \right\}$

c.) $-\frac{1}{2} e^{-t} \cdot t + \frac{1}{2} \operatorname{sen} t$

d.) $\mathcal{L}^{-1}\left\{ \dfrac{e^{-\pi s}}{s^2+16} \right\}$

d.) $\begin{cases} 0, & t < \pi \\ \frac{1}{4}\operatorname{sen}(4t), & t \geq \pi \end{cases}$

e.) $\mathcal{L}^{-1}\left\{ \operatorname{arctg}\left(\frac{4}{s}\right) \right\}$

e.) $\frac{1}{t} \operatorname{sen}(4t)$

f.) $\mathcal{L}^{-1}\left\{ \dfrac{1}{(s^2+1)^2} \right\}$

f.) $\frac{1}{2}\operatorname{sen} t - \frac{1}{2} t \cos t$

4.) Use o operador Transformada de Laplace para determinar as soluções das seguintes equações diferenciais que verifiquem as condições iniciais dadas:

Respostas

a.) $\begin{cases} y'' + 4y' + 4y = e^{-x}, & x \geq 0 \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$

R: $y = e^{-x} - e^{-2x}$

b.) $\begin{cases} y'' + 4y' + 3y = 0 & , \quad t \geq 0 \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$

R: $y = \frac{1}{2} e^{-t} - \frac{1}{2} e^{-3t}$

c.) $\begin{cases} y'' + 6y - 7 = 0 & , \quad t \geq 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$

R: $y = \frac{7}{6} - \frac{1}{6} \cos(\sqrt{6}t)$

d.) $\begin{cases} y'' - y' - 2y = x & , \quad x \geq 0 \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

R: $y = \frac{1}{4} - \frac{1}{2}x - \frac{1}{3} e^{-x} + \frac{1}{12} e^{2x}$

e.) $\begin{cases} y^{(iv)} - K^4 y = 0 & , \quad t \geq 0 \\ y(0) = 1 \\ y'(0) = y''(0) = y'''(0) = 0 \end{cases}$

R: $y = \frac{1}{4} e^{Kt} + \frac{1}{4} e^{-Kt} + \frac{1}{2} \cos(Kt)$

f.) $y' + 2y = e^{t} \quad , \quad t \geq 0$

R: $y = C_1 e^{-2t} + \frac{1}{3} e^{t}$

g.) $y'' + 4y' + 3y = 0 \quad , \quad t \geq 0$

R: $y = C_1 e^{-3t} + C_2 e^{-t}$