

2ª LISTA DE EDPS

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A) Verifique se as séries abaixo convergem ou divergem, justificando convenientemente:

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|---|---|
| 1) $\sum_{n=1}^{\infty} \frac{n+1}{n(n+2)}$ | 17) $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{\sqrt[n]{n^n}}$ |
| 2) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ | 18) $\sum_{n=1}^{\infty} \left(\frac{-1}{n}\right)^n$ |
| 3) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$ | 19) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ |
| 4) $\sum_{n=3}^{\infty} \frac{n}{n^2+4}$ | 20) $\sum_{n=1}^{\infty} \frac{e^{3n}}{(n+1)!}$ |
| 5) $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln^3(\sqrt[n]{n})}$ | 21) $\sum_{n=1}^{\infty} \frac{2^n}{(n+2)!}$ |
| 6) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$ | 22) $\sum_{n=1}^{\infty} \frac{n}{(2n)!}$ |
| 7) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2\sqrt{n}}$ | 23) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ |
| 8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} - \sqrt{n}}$ | 24) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+2}$ |
| 9) $\sum_{n=1}^{\infty} \frac{n}{n^4 + 2\sqrt{n}}$ | 25) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ |
| 10) $\sum_{n=1}^{\infty} \frac{n}{n^3 + 5}$ | 26) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ |
| 11) $\sum_{n=1}^{\infty} \frac{n^{9n+1}}{4^{n-4}}$ | 27) $\sum_{n=1}^{\infty} \frac{3 + \cos(n\pi)}{n^2}$ |
| 12) $\sum_{n=1}^{\infty} \frac{7^{n+3}}{n^{3n}}$ | 28) $\sum_{n=1}^{\infty} \operatorname{sen}^{10}\left(\frac{7n!}{n+2}\right) \cdot \frac{3}{n^3}$ |
| 13) $\sum_{n=1}^{\infty} \frac{e \cdot n}{n^{2n} \cdot 2}$ | 29) $\sum_{n=1}^{\infty} \cos^4\left(\frac{\sqrt{n}\pi + 4}{n-5}\right) \cdot \frac{2}{n^{3/2}}$ |
| 14) $\sum_{n=1}^{\infty} \left(\frac{n}{2+n^2}\right)^{5n}$ | 30) $\sum_{n=1}^{\infty} \sqrt[3]{\operatorname{sen}((3n+1)\pi)} \cdot \frac{4}{n^5}$ |
| 15) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^{3n}}$ | 31) $\sum_{n=2}^{\infty} \frac{2 - \operatorname{sen}((3n+1)\pi)}{\sqrt[5]{n^2}}$ |
| 16) $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n}$ | 32) $\sum_{n=2}^{\infty} \frac{3 + \cos((3n+1)\pi)}{\sqrt[5]{n^7}}$ |

B) Determine se a série é absolutamente convergente, condicionalmente convergente ou divergente, justificando convenientemente:

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| 1) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ | 7) $\sum_{n=1}^{\infty} \frac{(-1)^n 7}{n \cdot 2}$ |
| 2) $\sum_{n=1}^{\infty} \frac{(-1)^n \lfloor \ln(n) \rfloor}{n}$ | 8) $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$ |
| 3) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 5}{n^3 + 2}$ | 9) $\sum_{n=1}^{\infty} (-1)^n \cdot e^{-n}$ |
| 4) $\sum_{n=3}^{\infty} \frac{(-1)^n n}{n^2 + 9}$ | 10) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ |
| 5) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot e^{2n} + 1}{e^{2n}}$ | 11) $\sum_{n=1}^{\infty} (-1)^n \cdot (1 + e^{-n})$ |
| 6) $\sum_{n=1}^{\infty} (-1)^n \cdot (n \cdot 5^{-n})$ | 12) $\sum_{n=1}^{\infty} \frac{n!}{(-5)^n}$ |

C) Determine o intervalo de convergência e o raio de cada série de potência abaixo:

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| 1) $\sum_{n=0}^{\infty} \frac{1}{n+4} x^n$ | 10) $\sum_{n=1}^{\infty} \frac{n+1}{6^n} (x-4)^{\frac{n}{3}}$ |
| 2) $\sum_{n=0}^{\infty} \frac{x^n}{n^2+4}$ | 11) $\sum_{n=1}^{\infty} \frac{2}{(-4)^n} x^{2n}$ |
| 3) $\sum_{n=0}^{\infty} \frac{n^2}{2^n} x^n$ | 12) $\sum_{n=1}^{\infty} \frac{n!}{1000^n} \cdot (x+21)^n$ |
| 4) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{\sqrt{n}}$ | 13) $\sum_{n=1}^{\infty} n! \cdot x^n$ |
| 5) $\sum_{n=1}^{\infty} \frac{1}{2n+14} (x+3)^n$ | 14) $\sum_{n=1}^{\infty} \frac{8^n}{3^{2n}} x^n$ |
| 6) $\sum_{n=1}^{\infty} \frac{n^n}{3^{2n}} (x-2)^{5n}$ | 15) $\sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{\sqrt{n}}$ |
| 7) $\sum_{n=1}^{\infty} \frac{1}{4n\sqrt{n}} x^n$ | 16) $\sum_{n=1}^{\infty} \frac{10^n}{(3n+1)!} (x-3)^n$ |
| 8) $\sum_{n=1}^{\infty} \frac{(n+1)}{n!} (x-2)^n$ | 17) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n!} (x-4)^n$ |
| 9) $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!} x^{3n}$ | 18) $\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{n^n} (x+6)^n$ |

D) Encontre a série que representa $f'(x)$ e determine seu raio de convergência:

$$1) f(x) = \sum_{n=0}^{\infty} \frac{1}{n+4} x^n$$

$$2) f(x) = \sum_{n=0}^{\infty} \frac{2}{(2n)!} x^{3n}$$

$$3) f(x) = \sum_{n=0}^{\infty} (n+1)(x-1)^n \cdot 5^n$$

$$4) f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n16^n} x^{2n+1}$$

$$5) f(x) = \sum_{n=2}^{\infty} (x-4)^{2n-3} \cdot (2n-2)!$$

E) Sabendo que $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$, encontre a série de potências que representa $\sin x$ e seu raio de convergência.

F) Seja $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Encontre o intervalo de convergência das séries de potências de $f(x)$, $f'(x)$ e $f''(x)$.

G) Represente as funções abaixo como série de Taylor ou de Maclaurin em x_0 dado e determine o intervalo de convergência da série de potências obtida:

1) $f(x) = e^x$ como série de Taylor com $x_0 = -3$

2) $f(x) = e^{-x}$ como série de Maclaurin ($x_0 = 0$)

3) $f(x) = e^{4x}$ como série de Taylor com $x_0 = -2$

4) $f(x) = \ln x$ como série de Taylor com $x_0 = 1$

5) $f(x) = \frac{1}{1+2x}$ como série de Maclaurin ($x_0 = 0$)

6) $f(x) = \frac{1}{(1+x)^2}$ como série de Taylor com $x_0 = 1$

7) $f(x) = 7^x$ como série de Maclaurin ($x_0 = 0$).